Christ Church Grammar School

Mathematics Department
Year 11 Mathematics Methods

Semester 1 Examination, 2019

Question/Answer Booklet

MATHEMATICS METHODS

UNIT 1

Section One:
Calculator Free

SOLUTIONS

Student Name: ____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	14	14	100	98	65
				Total	100

Instructions to candidates

1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free **35% (52 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

Solve the following equations for x.

(a) $(5x - 3)(x + 4) = 0$. (1 mark)

Solution
$x = \frac{3}{5}, \qquad x = -4$
Specific behaviours
✓ both correct solutions

(b) $\frac{x}{2} = \frac{4x - 3}{3}$. (2 marks)

Solution
$3x = 8x - 6$ $5x = 6 \Rightarrow x = \frac{6}{5}$
Specific behaviours
✓ cross multiplies ✓ correct solution

(c) $2x^2 = 6x$. (2 marks)

Solution
$2x(x - 3) = 0$ $x = 0, \qquad x = 3$
Specific behaviours
✓ one correct solution ✓ both correct solutions

Question 2 **(5 marks)**

(a) A circle of radius 2 has its centre at the point $(1, -4)$. Determine the equation of the circle in the form $x^2 + y^2 = ax + by + c$. (3 marks)

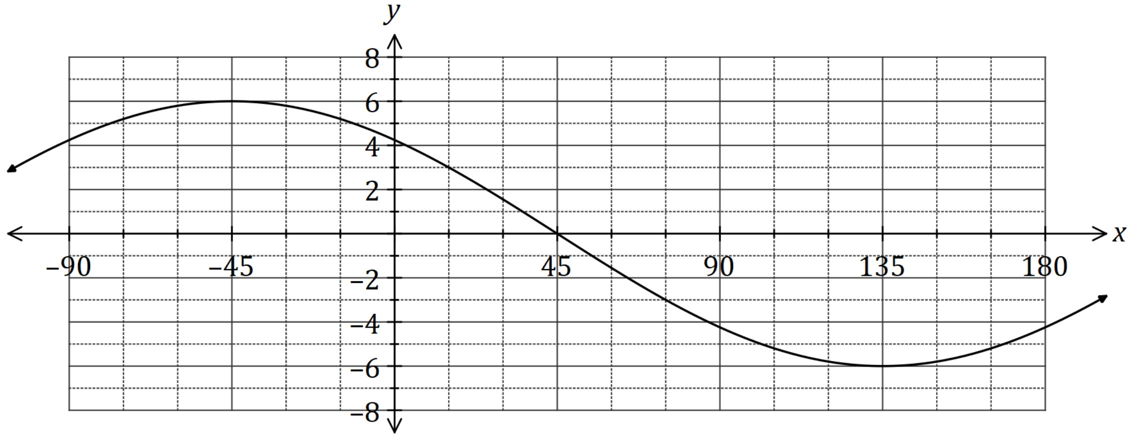
Solution
$(x-1)^2 + (y+4)^2 = 2^2$ $x^2 - 2x + 1 + y^2 + 8y + 16 = 4$ $x^2 + y^2 = 2x - 8y - 13$
Specific behaviours
✓ writes equation of circle ✓ correctly expands ✓ writes in required form

(b) The graph of $x = y^2$ passes through the point $(9, q)$. Determine the value(s) of q and hence explain why y is a relation but not a function of x. (2 marks)

Solution
$9 = q^2 \Rightarrow q = \pm 3$ A relation exists as we are told that $x = y^2$. The relation is not a function because it is not one-to-one (for most values of x there is more than one value of y).
Specific behaviours
✓ both possible values ✓ explains why relation not a function

Question 3 **(6 marks)**

(a) The graph of $y = a\cos(x + b)$ is shown below, where a and b are constants.



Determine the value of a and the value of b, where $-90° \le b \le 180°$. (2 marks)

Solution
$a = 6, \quad b = 45$
Specific behaviours
✓ value of a ✓ value of b

(b) Given that $0° \le x \le 360°$, solve

(i) $\cos(x) = \frac{1}{2}$. (1 mark)

Solution
$x = 60°, 300°$
Specific behaviours
✓ correct solutions

(ii) $8\cos(x + 30°) + 4\sqrt{3} = 0$. (3 marks)

Solution
$\cos(x + 30°) = -\frac{\sqrt{3}}{2}$ $x + 30° = 150°, 210°$ $x = 120°, 180°$
Specific behaviours
✓ simplifies equation ✓ solves for angle sum ✓ correct solutions

Question 4 **(7 marks)**

(a) Determine the coordinates of the

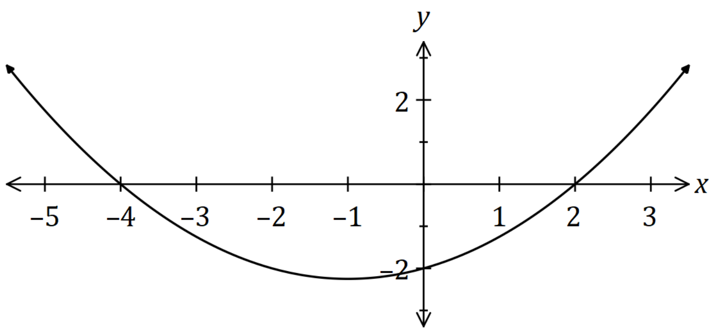
(i) y-intercept of the graph of $y = -2(x+4)^2 + 12$. (1 mark)

Solution
$x = 0, y = -2(4)^2 + 12 = -32 + 12 = -20$ At $(0, -20)$
Specific behaviours
✓ correct coordinates

(ii) turning point of the graph of $y = (x-3)(x+1)$. (2 marks)

Solution
$x = (3-1) \div 2 = 1$ $y = (1-3)(1+1) = -4$ At $(1, -4)$
Specific behaviours
✓ correct x-coordinate ✓ correct y-coordinate

(b) The graph of $y = ax^2 + bx + c$ is shown below. Determine the value of the coefficients a, b and c. (4 marks)



Solution
$y = a(x+4)(x-2)$ $-2 = a(4)(-2) \Rightarrow a = \frac{1}{4}$ $y = \frac{1}{4}(x^2 + 2x - 8)$ $a = \frac{1}{4}, \quad b = \frac{1}{2}, \quad c = -2$
Specific behaviours
✓ uses roots to write in factored form ✓ uses y-intercept to determine a ✓ expands quadratic ✓ states all coefficients

Question 5 **(7 marks)**

(a) Expand $x(x+5)^2$. (2 marks)

Solution
$x(x^2+10x+25) = x^3+10x^2+25x$
Specific behaviours
✓ expands quadratic correctly ✓ correct expansion

(b) Let $f(x) = x^3 + 2x^2 - 11x - 12$.

(i) Determine $f(-1)$. (1 mark)

Solution
$f(-1) = (-1)^3 + 2(-1)^2 - 11(-1) - 12$ $= -1 + 2 + 11 - 12$ $= 0$
Specific behaviours
✓ correct value

(ii) Solve $f(x) = 0$. (4 marks)

Solution
$x^3+2x^2-11x-12 = (x+1)(x^2+bx-12)$ $-11x = bx - 12x \Rightarrow b = 1$ $x^2+x-12 = (x+4)(x-3)$ $(x+1)(x+4)(x-3) = 0 \Rightarrow x = -4, -1, 3$
Specific behaviours
✓ uses (i) to write cubic as linear and quadratic factor ✓ determines entire quadratic factor ✓ factorises quadratic ✓ all correct solutions

Question 6 **(7 marks)**

(a) Describe the behaviour of the y values for each of the following graphs, given the behaviour of the x values:

(i) $y = x^4$, as $x \to \infty$. (1 mark)

Solution
$y \to \infty$
Specific behaviours
✓ describes correct behaviour

(ii) $y = (2 - x)^3$, as $x \to \infty$. (1 mark)

Solution
$y \to -\infty$
Specific behaviours
✓ describes correct behaviour

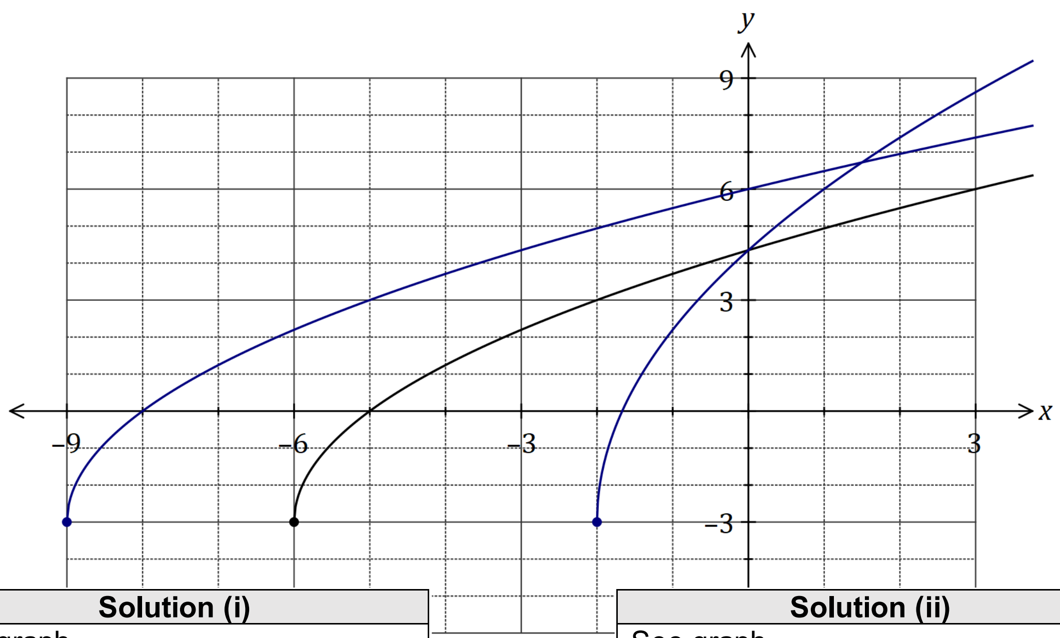
(iii) $y = \frac{1}{x}$, as $x \to -\infty$. (1 mark)

Solution
$y \to 0$
Specific behaviours
✓ describes correct behaviour

(b) The graph of $y = f(x)$ is shown below. On the same axes sketch the graph of

(i) $y = f(x + 3)$. (2 marks)

(ii) $y = f(3x)$. (2 marks)



Solution (i)
See graph
Specific behaviours
✓ smooth curve starting at $(-9, -3)$
✓ intercepts at $(-8, 0)$ and $(0, 6)$

Solution (ii)
See graph
Specific behaviours
✓ smooth curve starting at $(-2, -3)$
✓ same y-intercept as $f(x)$

Question 7 **(8 marks)**

(a) Complete the row of Pascal's triangle that starts $1, 5, 10, \ldots$ and express the sum of the numbers in this row as a power of 2. (2 marks)

Solution
$1, 5, 10, \mathbf{10}, \mathbf{5}, \mathbf{1}$ $\text{Sum} = 2^5$
Specific behaviours
✓ correct bolded terms ✓ correct power of 2

(b) Use ${}^{n}C_{r}$ notation to write down the seventh number in the row of Pascal's triangle that starts with 1, 8, 28, … (1 mark)

Solution
${}^{8}C_{2}$ or ${}^{8}C_{6}$
Specific behaviours
✓ correct answer using ${}^{n}C_{r}$ notation

(b) Determine the coefficient of

(i) the x^2 term in the expansion of $(7x-2)^2$. (1 mark)

Solution
$\binom{2}{2}(7x)^2(-2)^0 = 49x^2$ Coefficient is 49
Specific behaviours
✓ explicitly states coefficient

(ii) the x^4 term in the expansion of $(x+1)^5$. (1 mark)

Solution
$\binom{5}{4}(x)^4(1)^1 = 5x^4$ Coefficient is 5
Specific behaviours
✓ explicitly states coefficient

(iii) the x^3 term in the expansion of $(2-3x)^5$. (3 marks)

Solution
$\binom{5}{3}(2)^2(-3x)^3 = (10)(4)(-27)x^3$ $= -1080x^3$ Coefficient is: $-1\,080$
Specific behaviours
✓ indicates correct three factors of term ✓ expands each factor ✓ states coefficient

Question 8 **(7 marks)**

(a) Evaluate $\sin\left(\frac{39\pi}{4}\right)$. (2 marks)

Solution
$\sin\frac{39\pi}{4} = \sin\frac{(39-32)\pi}{4} = \sin\frac{7\pi}{4}$ $\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$
Specific behaviours
✓ reduces angle   ✓ exact value

(b) A is an acute angle and B is an obtuse angle such that $\cos A = \frac{1}{3}$ and $\sin B = \frac{2}{3}$.

(i) Show that $\sin A = \frac{2\sqrt{2}}{3}$ and determine the value of $\cos B$. (3 marks)

Solution
$\sin^2 A = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} \Rightarrow \sin A = \frac{\sqrt{8}}{3} = \frac{2\sqrt{2}}{3}$ $\cos^2 B = 1 - \left(\frac{2}{3}\right)^2 = \frac{5}{9}$ $\cos B = -\frac{\sqrt{5}}{3}$
Specific behaviours
✓ indicates how to obtain $\sin^2 A$ ✓ obtains $\cos^2 B$ ✓ correct value of $\cos B$

If right triangles are used, then:
✓uses Pythagoras' to show value of $\sin A$
✓obtains the absolute value of $\cos B$
✓ correct value of $\cos B$

(ii) Determine the value of $\sin(A + B)$ as a single fraction. (2 marks)

Solution
$\sin(A+B) = \frac{2\sqrt{2}}{3} \times \left(-\frac{\sqrt{5}}{3}\right) + \frac{1}{3} \times \frac{2}{3}$ $= \frac{2 - 2\sqrt{10}}{9}$
Specific behaviours
✓ substitutes correctly   ✓ correct value as single fraction

End of questions

Christ Church Grammar School

Mathematics Department
Year 11 Mathematics Methods

Semester 1 Examination, 2019

Question/Answer Booklet

MATHEMATICS METHODS

UNIT 1

Section Two:
Calculator Assumed

SOLUTIONS

Student Name: ____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on one unfolded sheet of A4 paper, and up to three calculators approved for use in this examination

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Section Two: Calculator-assumed **65% (98 Marks)**

This section has **fourteen (14)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(4 marks)**

(a) The points A and B have coordinates $(7, -2)$ and $(-3, 6)$ respectively. If A is the midpoint of B and C, determine the coordinates of C. (2 marks)

Solution
If $C(x, y)$ then $\frac{-3+x}{2} = 7 \Rightarrow x = 17$ and $\frac{6+y}{2} = -2 \Rightarrow y = -10$ $C(17, -10)$
Specific behaviours
✓ writes equations for midpoints or uses horizontal and vertical distance between points ✓ obtains the correct coordinates of C

(b) The points D and E have coordinates $(-2p, q)$ and $(3q, -2p)$ respectively, where p and q are constants. Determine the value of p and the value of q if the midpoint of D and E is at $(11, -7)$. (2 marks)

Solution
$\frac{-2p+3q}{2} = 11$ and $\frac{q-2p}{2} = -7$ Solve simultaneously CAS to get $p = 16, q = 18$
Specific behaviours
✓ value of p ✓ value of q

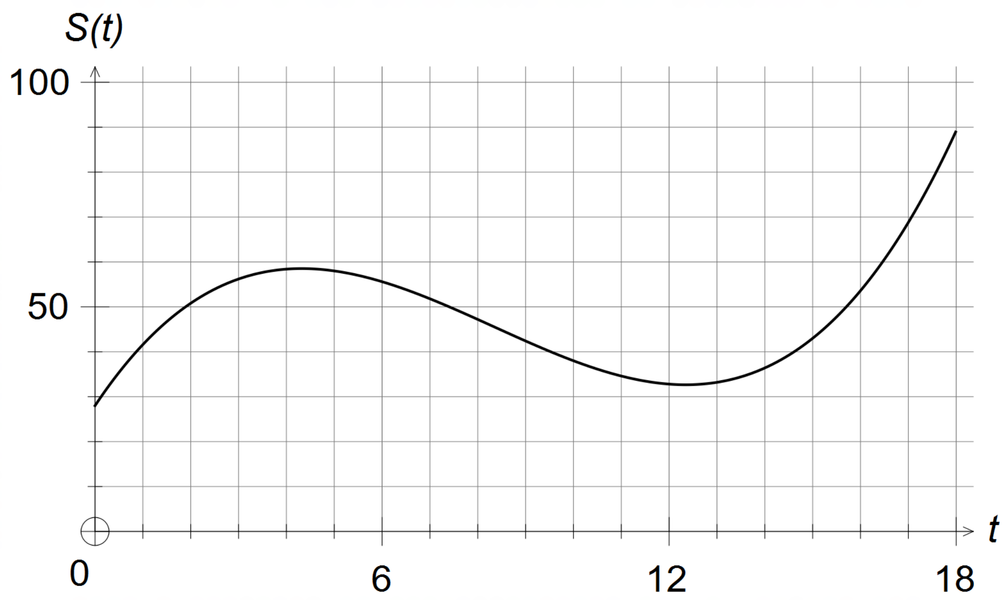
Question 10 **(8 marks)**

The average wind speed, $S(t)$ in km/h, over an 18 hour period from midnight to 6pm during a stormy day was observed to follow $S(t)=\frac{t^3}{10}-\frac{5t^2}{2}+16t+28$ where t was the number of hours since midnight.

(a) No data was available after 6pm as the measuring instrument broke at that time. What was the average wind speed at 6pm? (1 mark)

Solution
S(18) = 89.2 km/h
Specific behaviours
✓ states correct value

(b) On the grid below, sketch a graph to show how the average wind speed varied during the 18 hour period. (2 marks)



Solution
On grid above
Specific behaviours
✓ graph passes through (0, 28) approximately and (18, 89.2) approximately
✓ graph has correct shape

(c) At the height of the storm in the **<u>morning</u>**, some properties suffered structural and other damage. At what time, to the nearest quarter of an hour, did this occur? (2 marks)

Solution
t = 4.319 = 4hours and 19 mins So height of storm occurred at 4.15 am
Specific behaviours
✓ determines t value for maximum S(t)
✓ states correct time

Question 10 continued

(d) What was the lowest average wind speed recorded after 6am? (1 mark)

Solution
S(t) = 32.659 km/h
Specific behaviours
✓ states correct value

(e) For what percentage of the 18 hours did the average wind speed exceed 50 km/h? (2 marks)

Solution
From t = 1.89 to t = 7.40 and t = 15.70 to t = 18.00 = 5.51 hours + 2.30 hours = 7.81 hours 7.81/18 x 100 = 43.4%
Specific behaviours
✓ calculates the correct time periods ✓ calculates the correct percentage

Question 11 **(7 marks)**

Line L has equation $\frac{x}{5}+\frac{y}{3}=1$.

(a) State the coordinates of the point where L intersects the x-axis. (1 mark)

Solution
$y=0 \Rightarrow x=5$ At $(5,0)$
Specific behaviours
✓ correct coordinates (no marks if not listed as coordinates)

(b) State, with justification, if L is parallel to the line with equation $y=0.6x+4$. (2 marks)

Solution
$y=3-\frac{3}{5}x$ L is not parallel to this line as gradients are different: $-\frac{3}{5}\neq 0.6$.
Specific behaviours
✓ indicates gradient of L ✓ states not parallel, comparing gradients

(c) Determine the equation of line P that is perpendicular to L and passes through the point with coordinates $(30,19)$. (2 marks)

Solution
$y-19=\frac{5}{3}(x-30)$
Specific behaviours
✓ indicates perpendicular gradient ✓ correct equation

(d) Determine the coordinates of the point of intersection of L and P. (2 marks)

Solution
$P: y=\frac{5}{3}x-31$ $\frac{5}{3}x-31=3-\frac{3}{5}x \Rightarrow \frac{34}{15}x=34 \Rightarrow x=15$ $y=\frac{5}{3}(15)-31=-6$ Intersect at $(15,-6)$
Specific behaviours
✓ equates equations and solves for x ✓ correct coordinates

Award 2 marks if just the correct answer is listed.

Question 12 **(8 marks)**

(a) The variables Q and v are directly proportional and when $v = 40, Q = 10$.

(i) Determine an equation for the relationship between Q and v. (2 marks)

Solution
$Q = mv, \qquad m = \dfrac{10}{40} = 0.25 \Rightarrow Q = 0.25v$
Specific behaviours
✓ uses correct linear relationship ✓ calculates the proportion constant

v = 4Q is also correct and is to be awarded full marks

(ii) State the value of Q when $v = 80$. (1 mark)

Solution
$Q = 0.25(80) = 20$
Specific behaviours
✓ correct value

(b) The time, t minutes, that a car takes to travel 250 m at a constant speed of s kmh^{-1} is given by the formula $t = \dfrac{k}{s}$.

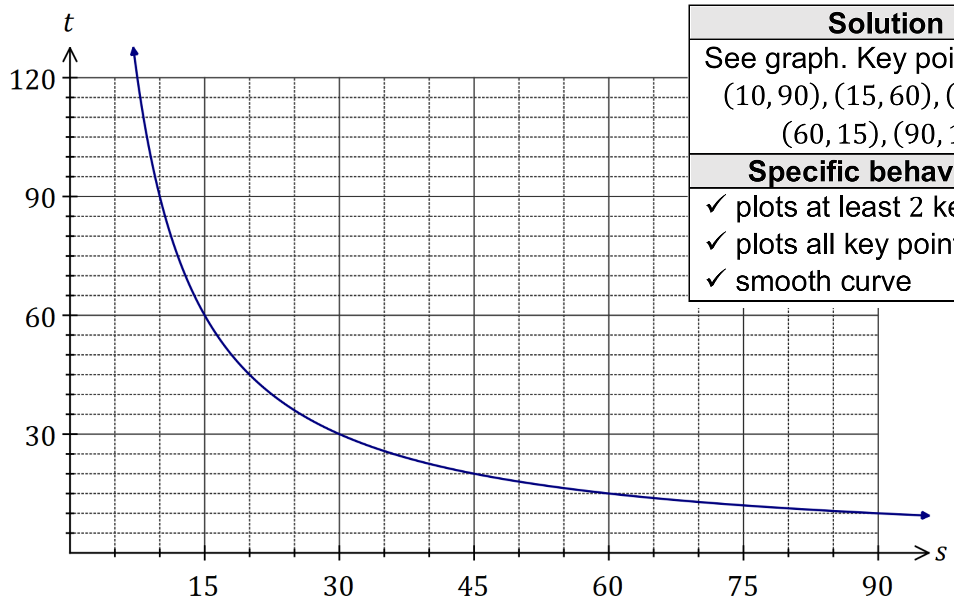
(i) Determine the value of the constant k, given that when $s = 15, t = 60$. (1 mark)

Solution
$60 = \dfrac{k}{15} \Rightarrow k = 900$
Specific behaviours
✓ correct value

(ii) Determine the value of t when $s = 10$. (1 mark)

Solution
$t = 900 \div 10 = 90$
Specific behaviours
✓ correct value

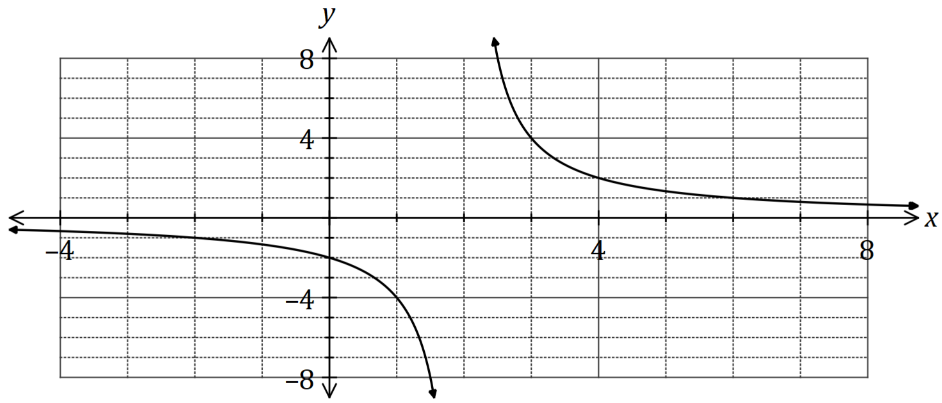
(iii) On the axes below, draw a graph to show how t varies with s. (3 marks)



Solution
See graph. Key points: $(10, 90), (15, 60), (30, 30), (60, 15), (90, 10)$
Specific behaviours
✓ plots at least 2 key points ✓ plots all key points ✓ smooth curve

Question 13 **(8 marks)**

The graph of $y = f(x)$ is shown below where $f(x) = \dfrac{a}{b - x}$.



(a) State the value of the constant a and the value of the constant b. (3 marks)

Solution
$b = 2$ Using $(3, 4) \Rightarrow 4 = a \div (2 - 3) \Rightarrow a = -4$
Specific behaviours
✓ value of b ✓ uses point on curve   ✓ value of a

(b) The hyperbola shown above has two asymptotes. State their equations. (2 marks)

Solution
$x = 2, \qquad y = 0$
Specific behaviours
✓ vertical asymptote   ✓ horizontal asymptote

(c) Describe how to transform the graph of $y = f(x)$ to obtain the graph of $y = f(x + 1)$ and state the domain and range of the transformed function. (3 marks)

Solution
Translate the graph 1 unit to left.   Domain: $x \neq 1$ Range: $y \neq 0$
Specific behaviours
✓ transformation   ✓ domain   ✓ range

Question 14 **(8 marks)**

(a) Convert, giving an exact answer

(i) $40°$ to radians. (1 mark)

Solution
$\frac{2\pi}{9}$
Specific behaviours
✓ exact value

(ii) 0.2 radians to degrees. (1 mark)

Solution
$\frac{36}{\pi}$
Specific behaviours
✓ exact value

(b) Calculate, to the nearest degree, the acute angle between the line $y = 4.5x + 2$ and the line $y = 1.5x - 3$. (3 marks)

Solution
$\theta_1 = \tan^{-1} 4.5 = 77.5$ $\theta_2 = \tan^{-1} 1.5 = 56.3$ $\theta_2 - \theta_1 = 21°$
Specific behaviours
✓ indicates use of $\tan^{-1} m$ ✓ one correct angle with x-axis   ✓ correct angle between lines

(c) The sides adjacent to the right-angle in a right triangle have lengths 36 cm and 77 cm.

If the smallest angle in the triangle is α, then determine an exact value for

(i) $\tan \alpha$. (1 mark)

Solution
$\tan \alpha = \frac{36}{77}$
Specific behaviours
✓ correct ratio

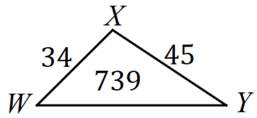
(ii) $\cos(90° - \alpha)$. (2 marks)

Solution
$\sqrt{36^2 + 77^2} = 85$ $\cos(90° - \alpha) = \frac{36}{85}$
Specific behaviours
✓ calculates length of hypotenuse   ✓ correct ratio

Question 15 **(7 marks)**

An **obtuse** angled triangle WXY has $w = 45$ cm, $y = 34$ cm and an area of 739 cm^2.

(a) Sketch a triangle to show this information. (1 mark)

Solution

Specific behaviours
✓ triangle with all information

(b) Determine the size of $\angle X$. (2 marks)

Solution
$739 = \frac{1}{2}(34)(45)\sin X$ $X = 104.98 \approx 105°$
Specific behaviours
✓ substitutes into area equation ✓ correct (obtuse) angle

(c) Show that $x \approx 63$ cm. (2 marks)

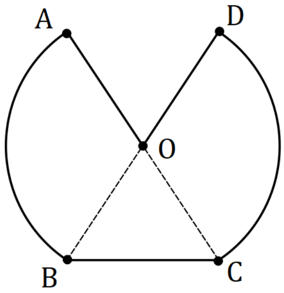
Solution
$x^2 = 34^2 + 45^2 - 2(34)(45)\cos 104.98$ $x = 63.02 \approx 63 \text{ cm}$
Specific behaviours
✓ uses appropriate equation that includes x ✓ substitutes correctly and solves to at least 1 dp

(d) Show that $\angle Y \approx 31°$. (2 marks)

Solution
$\frac{34}{\sin Y} = \frac{63.02}{\sin 104.98}$ $\angle Y = 31.4 \approx 31°$
Specific behaviours
✓ uses appropriate equation that includes Y ✓ substitutes correctly and solves to at least 1 dp

Question 16 **(7 marks)**

In shape $OABCD$ below, $\angle AOB = 117°$ and AC, BD are diameters of the circle with centre O and radius 42 cm.



(a) Calculate the perimeter of $OABCD$. (4 marks)

Solution
$117° = \frac{13\pi}{20}$, $\quad \pi - \frac{13\pi}{20} = \frac{7\pi}{20} = 63°$ $AB, DC: 42 \times \frac{13\pi}{20} = \frac{273\pi}{10} \approx 85.765$ $BC^2 = 42^2 + 42^2 - 2(42)(42)\cos 63$ $BC \approx 43.9$ $P_{TOTAL} = 2(42) + 2(85.765) + 43.9 \approx 299 \text{ cm}$
Specific behaviours
✓ indicates length of arc AB ✓ indicates use of cosine rule for BC ✓ correct length BC ✓ correct total perimeter

(b) Calculate the area of $OABCD$. (3 marks)

Solution
$AOB + DOC: 2 \times \frac{1}{2} \times 42^2 \times \frac{13\pi}{20} = \frac{5733\pi}{5} \approx 3602$ $BOC: \frac{1}{2}(42)(42)\sin 63 \approx 786$ $A_{TOTAL} = 3602 + 786 = 4\,388 \text{ cm}^2$
Specific behaviours
✓ sector area AOB ✓ triangle area BOC ✓ correct total area

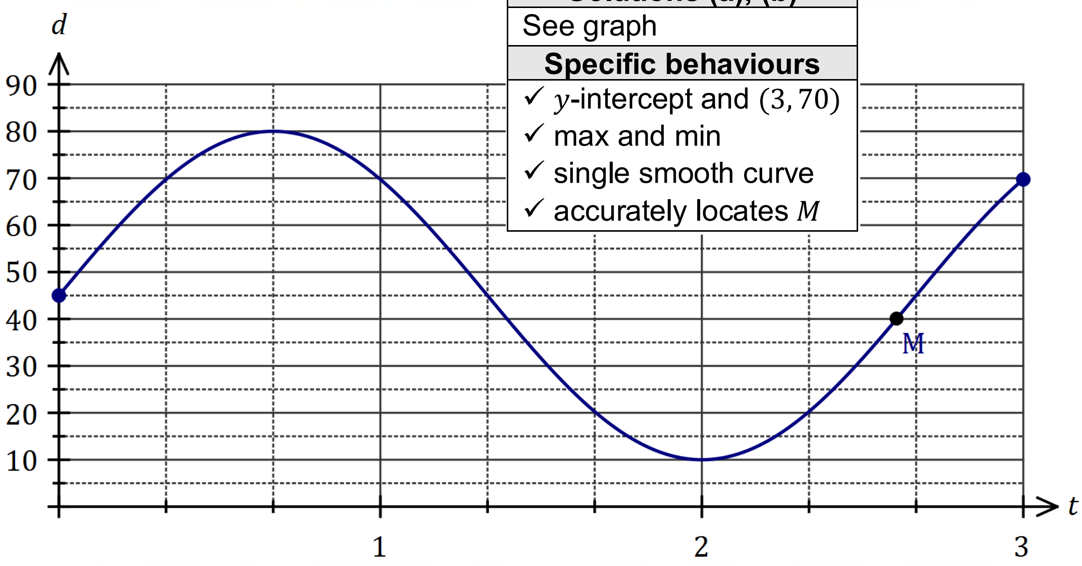
Question 17 **(7 marks)**

A small mass, attached to the bottom of a spring, oscillated up and down. The distance, d cm, of the mass from the top of the spring after t seconds can be modelled by

$$d = 45 + 35\sin\left(\frac{3\pi t}{4}\right)$$

(a) Sketch the graph on the axes below for $0 \le t \le 3$. (3 marks)

Solutions (a), (b)
See graph
Specific behaviours
✓ y-intercept and $(3, 70)$
✓ max and min
✓ single smooth curve
✓ accurately locates M



(b) Mark on your graph point M, where the mass is 40 cm from the top of the spring and moving downwards. (1 mark)

(c) Determine

(i) the maximum distance of the mass from the top of the spring. (1 mark)

Solution
80 cm
Specific behaviours
✓ correct distance

(ii) the time taken for the mass to first return to its initial position. (1 mark)

Solution
$t = \frac{4}{3} = 1.\overline{3}$ s
Specific behaviours
✓ correct time

(iii) the distance moved by the mass between $t = 1$ and $t = 2$. (1 mark)

Solution
$d(1) - d(2) = 69.75 - 10$ $= 59.75$ cm
Specific behaviours
✓ correct distance

Question 18 **(8 marks)**

(a) The equation of the axis of symmetry for the graph of $y = 2x^2 + 8x + 5$ is $x = m$. Determine the value of m, using a method that does not refer to the graph of the parabola. (2 marks)

Solution	
$x = -\dfrac{8}{2 \times 2} = -2$ $m = -2$	$y = 2(x+2)^2 + c$ $\therefore m = -2$
Specific behaviours	
✓ uses $x = -b \div (2a)$ or partially completes the square ✓ value of m	

(b) A parabola with equation $y = ax^2 + bx + c$ has a turning point at $(4, -5)$ and passes through the point $(2, -17)$. Determine the value of a, the value of b and the value of c. (3 marks)

Solution	
$y = a(x-4)^2 - 5$ $-17 = a(2-4)^2 - 5 \Rightarrow a = -3$ $y = -3(x-4)^2 - 5$ $= -3x^2 + 24x - 53$ $a = -3, \quad b = 24, \quad c = -53$	$-\dfrac{b}{2a} = 4$ $b = -8a$ $sub\ (4, -5)\ and\ (2, -17)\ gives$ $-5 = -16a + c$ $-17 = -12a + c$ $a = -3, \quad b = 24, \quad c = -53$
Specific behaviours	
✓ correctly writes in turning point form ✓ solves for a using point ✓ expands and states all values	✓ correctly uses axis of symmetry formula ✓ solves for 2 variables ✓ correctly solves all 3 variables

(c) Determine the value of the discriminant for the quadratic equation $4x^2 - 28x + 47 = 0$ and use it to explain how many solutions the equation $(x+3)(4x^2 - 28x + 47) = 0$ will have. (3 marks)

Solution
$d = (-28)^2 - 4(4)(47) = 32$ When $d > 0$, quadratic will have two solutions. Hence equation will have three solutions - one from the linear factor and two from the quadratic factor.
Specific behaviours
✓ value of discriminant ✓ uses discriminant to say quadratic will have two solutions ✓ explains why equation has three solutions

Question 19 **(6 marks)**

Let $p = \cos 130°$ and $q = \sin 35°$.

Give your answers to the following in terms of p and/or q.

(a) Write down an expression for

(i) $\sin 145°$. (1 mark)

Solution
$\sin 145° = \sin 35° = q$
Specific behaviours
✓ correct expression

(ii) $\cos 50°$. (1 mark)

Solution
$\cos 50° = -\cos 130° = -p$
Specific behaviours
✓ correct expression

(b) Determine an expression for $\cos 145°$. (3 marks)

Solution
$\cos^2 145° + \sin^2 145° = 1$ $\cos^2 145° = 1 - q^2$ $\cos 145° = \pm\sqrt{1-q^2}$ but $\cos 145° < 0 \Rightarrow \cos 145° = -\sqrt{1-q^2}$
Specific behaviours
✓ indicates use of $\sin^2\theta + \cos^2\theta = 1$ ✓ expression for $\cos^2 145°$ ✓ correct expression

(c) Determine an expression for $\tan 145°$. (1 mark)

Solution
$\tan 145° = \dfrac{\sin 145°}{\cos 145°} = -\dfrac{q}{\sqrt{1-q^2}}$
Specific behaviours
✓ correct expression

Question 20 **(9 marks)**

(a) Show, using one or more identities from the formula sheet and without using the value of any trigonometric term, that (3 marks)

$$\cos 15° \cos 65° + \sin 15° \sin 65° = \sin 140°$$

Solution
$\cos 15° \cos 65° + \sin 15° \sin 65° = \cos(65° - 15°)$ $= \cos 50°$ $= \sin(90° + 50°)$ $= \sin 140°$
Specific behaviours
✓ shows uses of difference identity ✓ reduces LHS to $\cos 50°$ ✓ shows use of relationships between trigonometric ratios to obtain result

(b) Simplify $\sin(A+B)\cos B - \cos(A+B)\sin(B)$. (2 marks)

Solution
$\sin(A+B)\cos B - \cos(A+B)\sin(B) = \sin(A+B-B)$ $= \sin A$
Specific behaviours
✓ indicates use of difference identity ✓ correct result

(c) (i) Show that $\sqrt{2}\sin(x+45°) = \sin x + \cos x$ (2 marks)

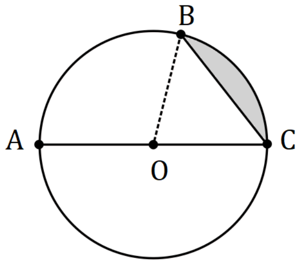
Solution
$\sqrt{2}\sin(x + 45°) = \sqrt{2}\sin x \cos 45° + \sqrt{2}\sin 45° \cos x$ $= \sin x + \cos x$
✓ correctly expands LHS ✓ concludes correctly

(ii) Hence, show that the exact value of $\sin 75° = \frac{(1+\sqrt{3})}{2\sqrt{2}}$ (2 marks)

Solution
$\sqrt{2}\sin 75° = \sqrt{2}\sin(45° + 30°) = \sin 30° + \cos 30°$ $= \frac{1}{2} + \frac{\sqrt{3}}{2}$ or $\frac{1}{2}(1 + \sqrt{3})$ Therefore $\sin 75° = \frac{(1+\sqrt{3})}{2\sqrt{2}}$
Specific behaviours
✓ substitutes x = 30 into expression in (c) (i) ✓ uses exact values to obtain correct conclusion

Question 21 **(8 marks)**

(a) The circle shown has centre O and diameter AC of length 50 cm. Determine the shaded area given that $2 \times \angle AOB = 3 \times \angle BOC$. (4 marks)



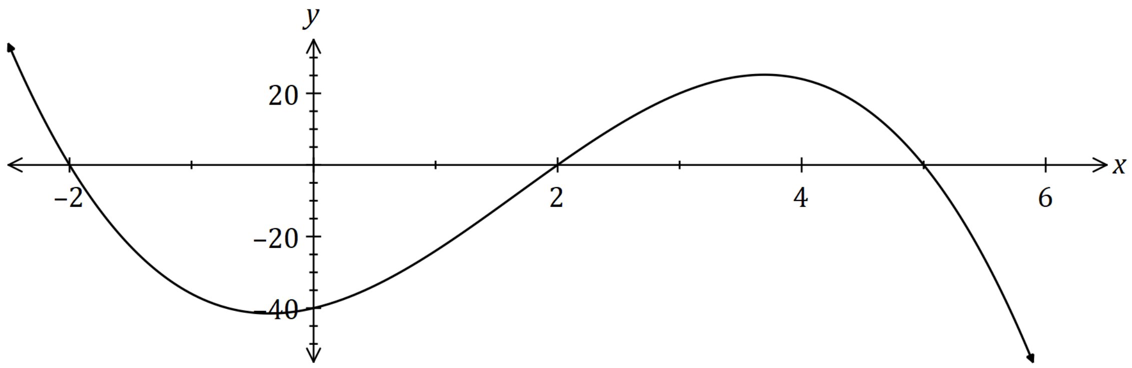
Solution
$\angle AOB = \frac{3}{2}\angle BOC$ $\angle BOC + \frac{3}{2}\angle BOC = \pi \Rightarrow \angle BOC = \frac{2\pi}{5} = 72°$ $A = \frac{1}{2}(25)^2\left(\frac{2\pi}{5} - \sin\frac{2\pi}{5}\right)$ $= 95.5\text{ cm}^2$
Specific behaviours
✓ equation using angles ✓ correct angle for segment ✓ substitutes correctly into formula ✓ correct area

(b) A sector of a circle has a perimeter of 112 cm and an area of 735 cm^2. Determine all possible values for the radius of the circle. (4 marks)

Solution
$2r + r\theta = 112$ $\frac{1}{2}r^2\theta = 735$ Solving simultaneously gives $r = 21, \theta = \frac{10}{3}$ or $r = 35, \theta = \frac{6}{5}$ Hence $r = 21$ or $r = 35$ cm
Specific behaviours
✓ equation for perimeter ✓ equation for area ✓ solution of equations ✓ states both values of r

Question 22 **(3 marks)**

The graph of the cubic function $y = f(x)$ is shown below. Determine $f(10)$.



Solution
$f(x) = a(x+2)(x-2)(x-5)$ $-40 = a(2)(-2)(-5) \Rightarrow a = -2$ $f(10) = -2(12)(8)(5) = -960$
Specific behaviours
✓ cubic in factored form ✓ correct value of a ✓ required value

End of questions